IV. Conclusion

The coupling terms in the perturbation formulas for both the waveguide and resonator perturbations are proportional to the coupling parameters χ and κ in the constitutive equations, and to the dot product of the electric and magnetic fields in unperturbed modes. In the present study we have demonstrated that by exciting properly two degenerate modes in waveguides or resonators with ideally conducting walls, the perturbation is proportional to either the nonreciprocity parameter χ or the chirality parameter κ. This gives a way to measure the material parameters χ and κ separately. Moreover, it appears that it is possible to distinguish between the effects of chirality and nonreciprocity by changing the phase shift of the two modes in a waveguide or a resonator. This makes the method rather convenient and simple.

If there exist only H- or E-polarized fields in a waveguide, the coupling term is always imaginary, and the nonreciprocity of inclusions gives only a second-order effect on the propagation factor. In contrast, in a resonator with either H- or E-modes, the coupling term is always real, and there are no first-order effects on the resonant frequency due to the chirality parameter κ. Physically, the perturbational methods of the biisotropic media parameters measurement are based on the coupling between two orthogonal modes when a biisotropic sample is present inside a waveguide or a resonator. Since the coupling effect is small, it seems preferable to excite both the modes by an external source and to measure the shift of the resonant frequency or the propagation factor of a degenerate mode. This approach makes the effect more pronounced. The paper presents a theoretical treatment of the measurement problem and there are many practical considerations to be taken into account. For example, the two degenerate modes are also coupled because of the losses in the walls of a waveguide or a resonator, and that can mask small effects due to the inclusion. These and other possible sources for errors have to be eliminated in practical applications of the suggested measurement techniques.

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An Efficient Method for Computing the Capacitance Matrix of Multiconductor Interconnects in Very High-Speed Integrated Circuit Systems

Shui-Ping Luo and Zheng-Fan Li

***Abstract*— A new method for computing the capacitance matrix of multiconductor interconnects with finite metallization thickness is developed. Converting the vertical wall of the rectangular conductors into the equivalent horizontal strips allows the Green's function in the spectral domain and the FFT algorithm to be used, which makes the method more effective for computing capacitance matrix of the interconnects.**

I. Introduction

As is well known, the computation of the electrical parameters of the interconnects for very high-speed integrated circuit systems is a hard task, even with the quasi-TEM assumption. For such structures (for example, the chip-to-chip or on-chip interconnects for VHSIC), multiconductor transmission lines with finite metallization thickness in multilayered dielectric media are used. C. Wei, R. F. Harrington, and others, [1], [2] have employed the well known moment method using the total charge Green's function, which has a very simple form (like the free space Green's function). But the method is consuming

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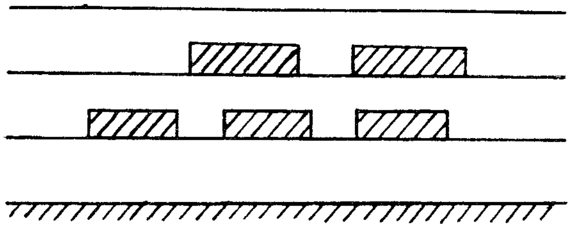

Fig. 1. An interconnect system.

of computer resources due to additional subsections on the dielectric-to-dielectric interface in the moment method. Other methods, such as the spectral domain method and the method of lines, are effective only for the case of zero-thickness conductors. This paper presents an effective but simple method for calculating the capacitance matrix of the interconnects considering their finite metallization thickness. The results from this method are sufficiently accurate for application and are in agreement with those of others, but the computing efficiency of this method is improved.

II. Theory and Analysis

The cross-section of the interconnects discussed here is shown as Fig. 1. The structure consists of a multiconductor with rectangular shape and some planar dielectric layers. Then, we divide the contour of all conductors into N equal-length subsections to obtain a discrete system. According to the moment method with rectangular pulse basis functions and point-matching test functions, the potential functions at the mid-points of each subsection can be presented as

$$\Phi_{m,n} = \sum_{n=1}^{N} g_{m,n}\sigma_n \qquad m = 1, 2, \cdots, N \tag{1}$$

where σ_n is the average charge density to be determined on n-th subsection and $g_{m,n}$ is the coefficient given by

$$g_{m,n} = \int_{\Delta l m} T_m(x, y) \int_{\Delta l n} B_n(x', y') G(x, y \mid x', y')\, dl'\, dl \tag{2}$$

where $B_n(x, y)$ is the basis function, $T_m(x, y)$ is the test function and $G(x, y \mid x', y')$ is the Green's function in the spatial domain.

As everyone knows, the expressions of free-charge Green's function in spatial domain for multilayered dielectric structure are tedious, and the convolution calculation in (2) is also quite complicated. Hence, the moment method using the general free charge Green's function in the spatial domain is lengthy in application.

If the free charges are located on the zero-thickness conducting strips between the dielectric layers, then the simple form Green's function in the spectral domain can be used and the convolution calculation can be replaced by multiplication in spectral domain. Then, the FFT algorithm is used for transformation between the spectral domain and spatial domain. This can apparently reduce the computing task.

For resolving the problems with finite-thickness conductor, the equivalence shown in Fig. 2 is used. Here, the subsections along the vertical wall of the conductors are rotated by 90 degrees around their centers to become the corresponding horizontal subsections, and the original structure of interconnects is replaced by the equivalent multilayered and horizontal metallic strips with zero-thickness. This equivalence is only valid for such interconnects, with gaps between the different conductors larger than their thickness. This is true for normal interconnects structures.

For the equivalent structure, the total length L of each layer existing on the metallic strips is divided into M equal subsections and their length is Δx. We then define

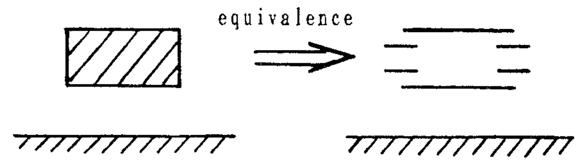

Fig. 2. The equivalent transforming of the rectangular conductor.

$$B_j(x) = \begin{cases} 1 & j\Delta x - \Delta x/2 < x < j\Delta x + \Delta x/2 \\ 0 & \text{elsewhere} \end{cases}$$
$$j = 0, 1, \cdots, M-1 \tag{3}$$

$$T_i(x) = \delta(x - i\Delta x) \qquad i = 0, 1, \cdots, M-1. \tag{4}$$

Here, $B_j(x)$ and $T_i(x)$ are the rectangular pulse basis function on jth subsection and Dirac test function on ith subsection, respectively.

The coefficient $g_{l,k,i,j}$ in (2) for ith subsection of lth layer to jth subsection of kth layer can be written as

$$\begin{aligned} g_{l,k,i,j} &= \int_0^L T_i(x) \int_0^L B_j(x') G_{l,k}(x \mid x')\, dx'\, dx \\ &= \int_0^L \delta(x - i\Delta x) \int_0^L B_0(x' - j\Delta x) G_{l,k}(x \mid x')\, dx'\, dx \\ &= B_0(x) * G_{l,k}(x) \end{aligned} \tag{5}$$

$G_{l,k}(x \mid x')$ is the Green's function for lth to kth metallic layer.

Transforming (5) from spatial domain into spectral domain, we can get following equation in spectral domain:

$$\tilde{g}_{l,k,i,j}(\alpha) = \tilde{B}_0(\alpha)\tilde{G}_{l,k}(\alpha) \tag{6}$$

where α is the variable in spectral domain and

$$\tilde{B}_0(\alpha) = F[B_0(x)] = \sin(\pi\alpha * \Delta x)/(\pi\alpha). \tag{7}$$

The symbol F means the Fourier transform.

$\tilde{G}_{l,k}(\alpha)$ is the Green's function in spectral domain for multilayered dielectric media, which can be easily obtained from the recurrence formulas in [3], so that coefficient $g_{l,k,i,j}$ in spatial domain can be obtained through inverse Fourier transform

$$g_{l,k,i,j} = F^{-1}[B_0(\alpha)G_{l,k}(\alpha)]|_{x=(i-j)\Delta x}. \tag{8}$$

Then, the capacitance matrix can be derived from $g_{l,k,i,j}$.

The Fourier transform is realized by the FFT algorithm. According to the sampling theorem, the number of the sample points M (i.e. the number of subsections on each layer) must be many enough to reflect the variation of the potential function on the horizontal strips and gaps for avoiding the aliasing. In our experience, 2–3 subsections along the vertical wall and 5–10 subsections along the horizontal wall on each conductor are sufficient for desirable computing accuracy. For the same reason, the total length L of each layer must be as large as 5–6 times of the range occupied by the conductors.

III. Numerical Examples

Example 1: A pair of coupled microstrips on a dielectric slab over a conductor ground plane ar shown in Fig. 3. If we take 2 subsections along the vertical wall, 6 subsections along the horizontal wall on each conductor, and 128 sample points along the total horizontal length L for FFT algorithm, the results from this method are agreeable to those of [1] and [2], as shown in Table I. However, the CPU time of this method is only about 1/5 of that of [2].

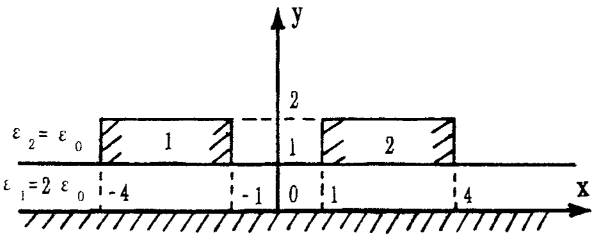

Fig. 3. Coupled microstrip.

TABLE I

capacitance F/m	this method	reference [1]	reference [2]
C(1, 1)	0.9213E-10	0.9165E-10	0.9236e-10
C(1, 2)	-0.8302E-11	-0.8220E-11	-0.8494E-11
C(2, 1)	-0.8302E-11	-0.8220e-11	-0.8494E-11
C(2, 2)	0.9213E-10	0.9165E-10	0.9236E-10

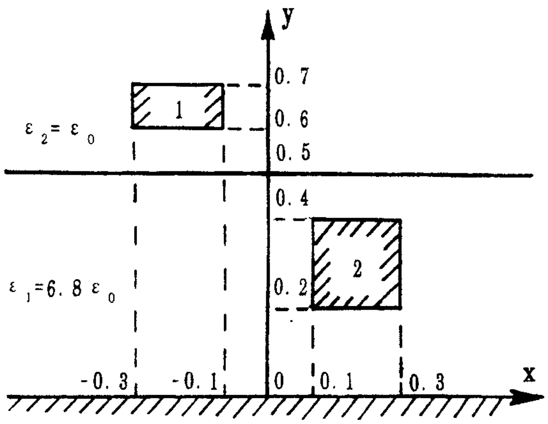

Fig. 4. Two conductors in two different dielectric layers.

TABLE II

capacitance F/m	this method	reference [1]	reference [4]
C(1, 1)	0.3697E-10	0.3651E-10	0.3701E-10
C(1, 2)	-0.1584E-11	-0.1562E-11	-0.1520E-11
C(2, 1)	-0.1584E-11	-0.1562E-11	-0.1520E-11
C(2, 2)	0.2134E-9	0.2099E-10	0.2108E-9

Example 2: There are two different rectangular conductors in two dielectric layers above a ground plane as shown in Fig. 4. The results using this method together with those of [1] and [4] are shown in Table II, and the computing speed of this method is also much faster than those of other methods.

IV. Conclusion

A new method for calculating the capacitance matrix of the multiconductor interconnects is given. The computing speed is faster than that of other methods with the same accuracy, and the desired storage of the computer is also decreased, so this method is effective for computing the electrical parameters of the interconnects for high-speed/high-complexity electronic systems.

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Eigenmode Sequence for an Elliptical Waveguide with Arbitrary Ellipticity

Shan-jie Zhang and Yao-chun Shen

***Abstract*—Eigenmode sequence for an elliptical waveguide with arbitrary ellipticity is studied by directly calculating the parametric zeros of the modified Mathieu functions of the first kind and their derivatives. The normalized cutoff wavelength of the lowest 100 successive modes are presented, and the curvefitting expressions for the determination of the cutoff wavelength of the lowest 10 order modes are given, which are valid for the ellipticities ranging from 0.0 to 0.99.**

I. Introduction

Elliptical waveguides have wide applications such as radar feed lines, multichannel communication and accelerator beam tubes. The determination of the cutoff wavelength of the elliptical waveguide is one of the most important problems for designing the waveguide or analyzing the wave propagation in the waveguide. In 1938, Chu [1] first presented the theory of the transmission of the electromagnetic waves in elliptical waveguide. Since then some more numerical results about the cutoff wavelengths in elliptical waveguide have been obtained [2]–[4]. In 1970, Kretzschmar [5] obtained the curves of the cutoff wavelengths for the 19 successive modes and the approximative formula for the eight lowest order modes. Recently Goldberg [6] calculated the cutoff wavelengths for the six lowest modes and gave a correction to the field pattern plotted in [1]. In fact, the determination of the cutoff wavelength of an elliptical waveguide is a problem of calculating the zeros of the modified

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